



# Joint modeling of biomarkers dynamics and survival with competing risks to predict the prognosis of patients hospitalized with severe infectious diseases

**Alexandra Lavalley-Morelle<sup>\*1</sup>, France Mentré<sup>1,2</sup>, Emmanuelle Comets<sup>1,3</sup>, Jimmy Mullaert<sup>1,2</sup>**

\*alexandra.lavalley-morelle@inserm.fr

1 Université Paris Cité, UMR 1137 IAME, INSERM, F-75018 Paris, France

2 Department of Epidemiology, Biostatistics and Clinical Research, AP-HP, Bichat-Claude Bernard University Hospital, F-75018 Paris, France

3 Université de Rennes, Inserm, EHESP, Irset - UMRS 1085, F-35000 Rennes, France

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Provide every patient with a self and adapted medical treatment

**Personalized medicine**

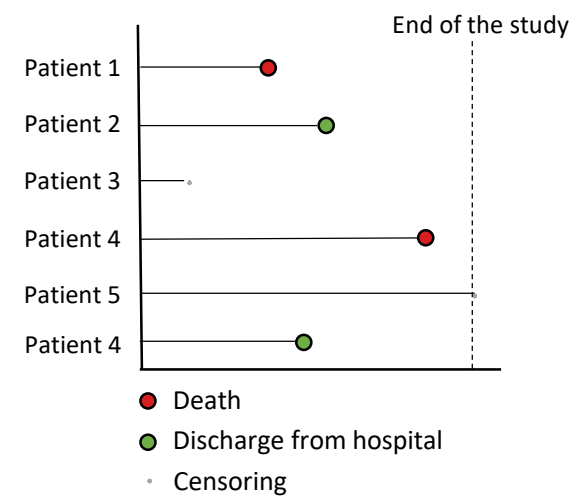


Provide every patient with a self and adapted medical treatment

# Personalized medicine



Predict the death of hospitalized patients for severe infectious diseases



Competing risks





Provide every patient with a self and adapted medical treatment



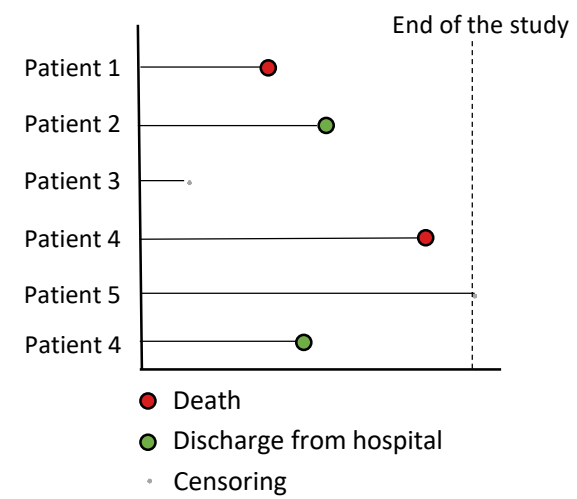
Predict the death of hospitalized patients for severe infectious diseases



Hospitals equipped with laboratory information systems that routinely gather results of biological analyses

# Personalized medicine

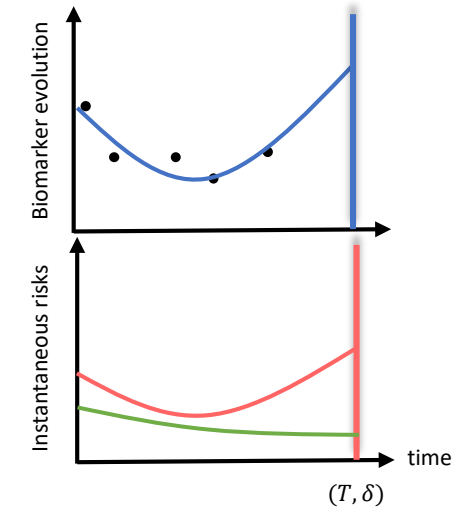
Consecutive biological observations can be used in a joint model to provide individual dynamic predictions<sup>1</sup> of patient prognosis



Competing risks

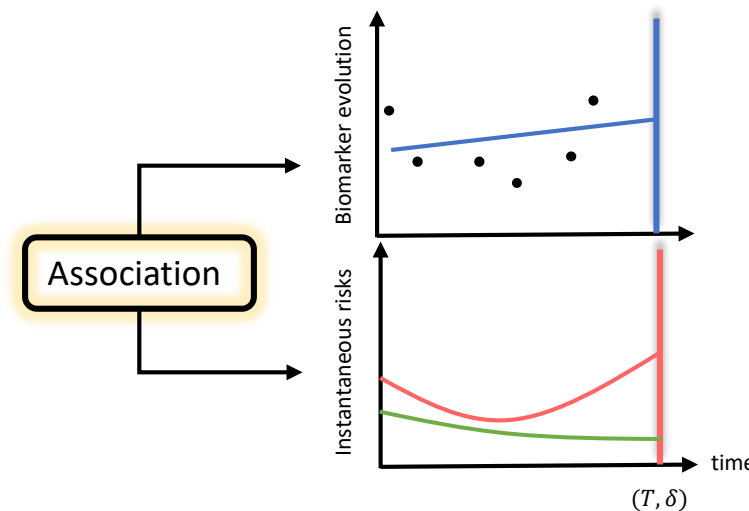
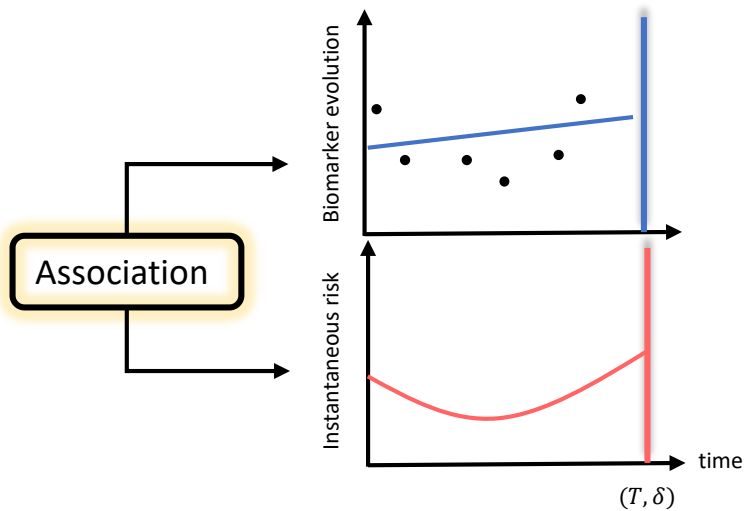


Association



Joint modeling of **LMEM** and **single event**

Joint modeling of **LMEM** and **competing risks**



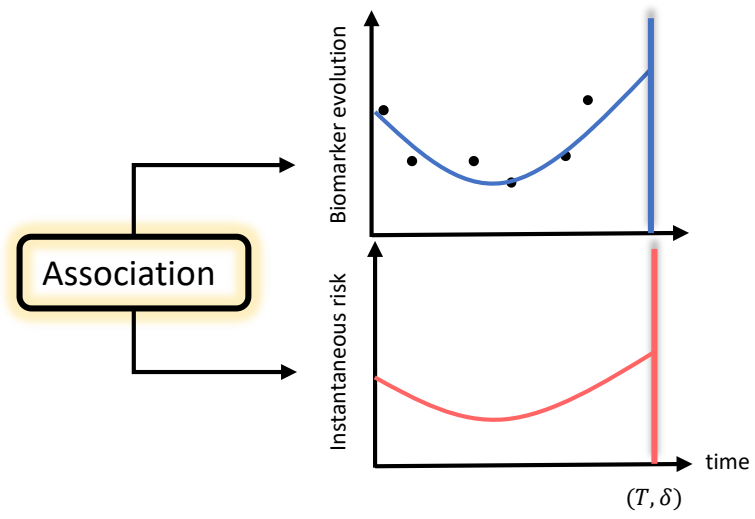
Widely developed in literature (single event<sup>1,2,3</sup>, competing risks<sup>4,5,6</sup>)  
Estimation available in various software: (R, SAS, Monolix, NONMEM,...)

**LMEM = linear mixed-effects model**

1- Rizopoulos. *Biometrics*, 2011  
 2- Angeli et al. *The AAPS Journal*, 2016  
 3- Elashoff et al. *Biometrics*, 2008

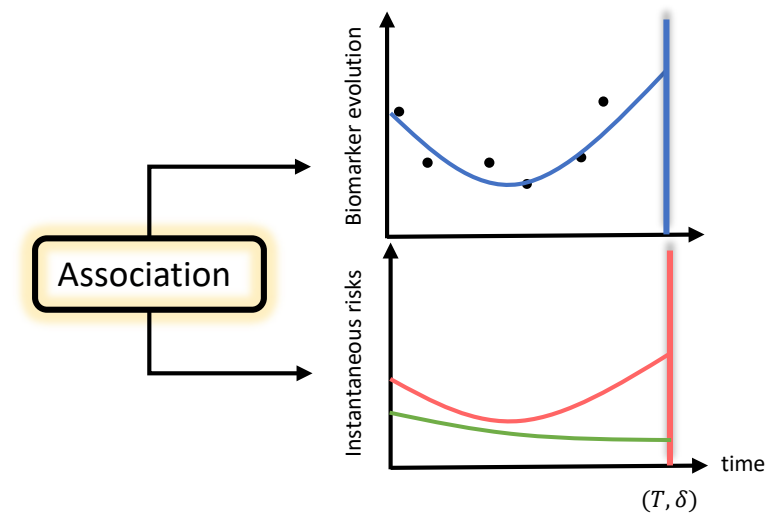
4- Deslandes and Chevret. *BMC Medical Research and Methodology*, 2010  
 5- Musoro et al. *Statistica Neerlandica*, 2018  
 6- Alvares and Rubio. *Statistics in Medicine*, 2021

Joint modeling of **NLMEM** and **single event**



Widely developed in literature<sup>7,8,9</sup>  
Some softwares available: Monolix, NONMEM

Joint modeling of **NLMEM** and **competing risks**



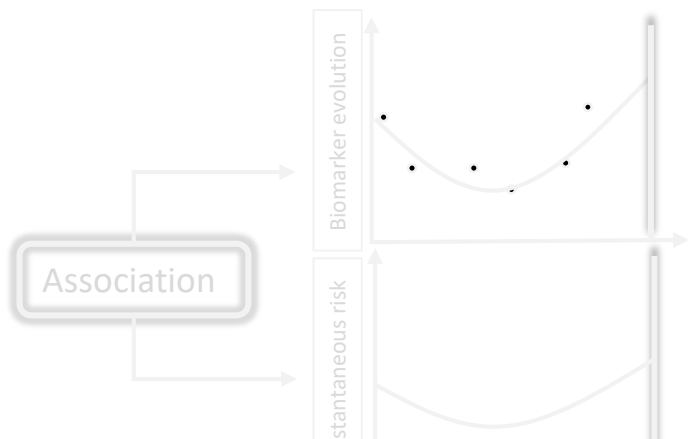
Very few developed in literature (1 published work<sup>10</sup>)  
Software used: NONMEM

**NLMEM = nonlinear mixed-effects model**

7- Desmée et al. *The AAPS Journal*, 2015  
8- Tardivon et al. *Clinical Pharmacology & Therapeutics*, 2019  
9- Kerioui et al. *Statistics in Medicine*, 2020

10- Krishnan et al. *CPT:Pharmacometrics & Systems Pharmacology*, 2021

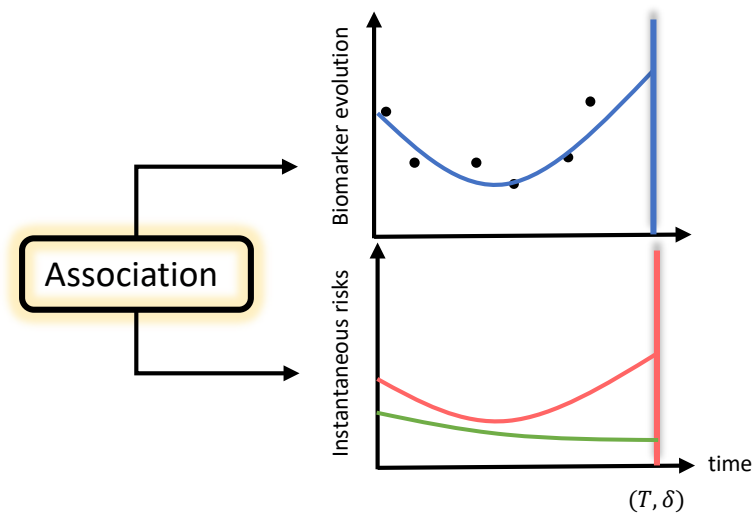
Joint modeling of NLMEM and single event



- 1st PhD work:
- Show that estimation well performs in **Monolix** software
  - Application on sepsis patients

- Lavalley-Morelle et al. *CPT:Pharmacometrics & Systems Pharmacology*, 2022
- 29th PAGE conference (2021), "Individual dynamic predictions and their evaluation in joint analysis with nonlinear longitudinal model and parametric competing risks model: application on sepsis patients"

Joint modeling of NLMEM and competing risks



Very few developed in literature (1 published work<sup>10</sup>)  
Software used: NONMEM

7- Desmée et al. *The AAPS Journal*, 2015  
8- Tardivon et al. *Clinical Pharmacology & Therapeutics*, 2019  
9- Kerioui et al. *Statistics in Medicine*, 2020

10- Krishnan et al. *CPT:Pharmacometrics & Systems Pharmacology*, 2021

Comes with computational and identifiability issues due to the high number of random effects<sup>11</sup>

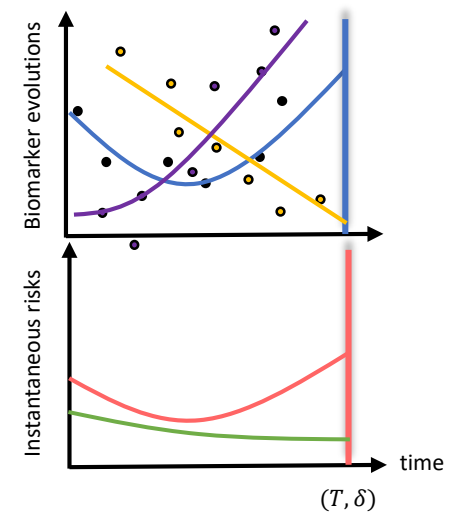
Published models mostly limited to longitudinal models with at most two biomarkers<sup>12,13</sup>

Certainly improve prediction accuracy

**Shifting the modeling from one to several biomarkers**

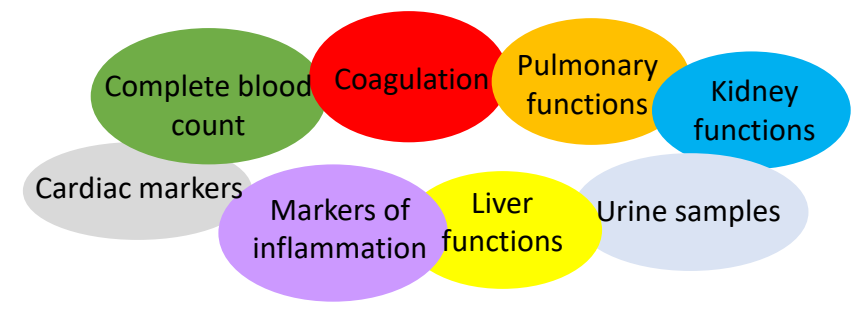
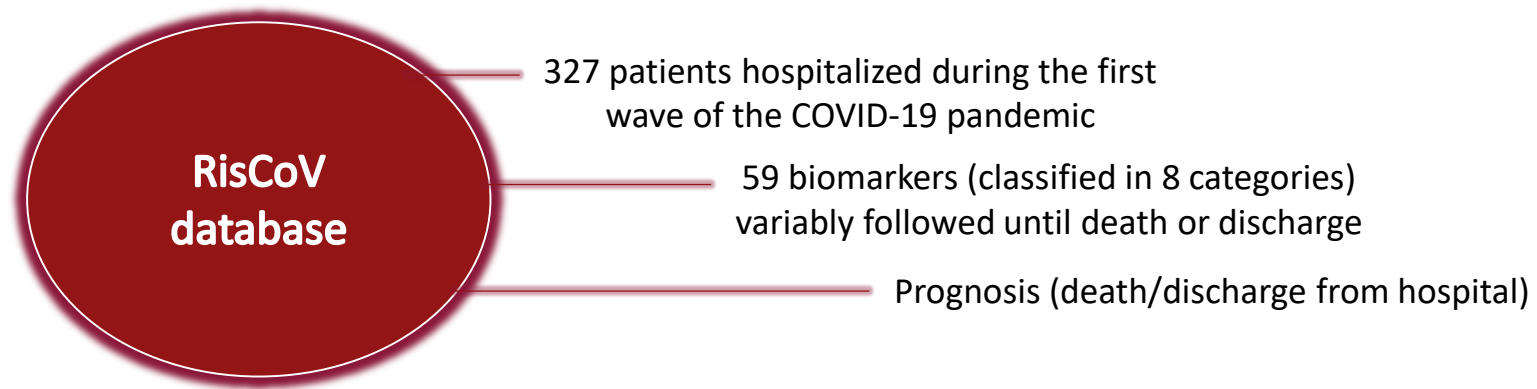
Need of methods for variable selection in this context

Association



11- Shen et al. *Statistics in Medicine*, 2021  
 12- Long and Mills. *BMC Medical Research Methodology*, 2018  
 13- Rajeswaran et al. *Statistics in Biosciences*, 2018





**How to select biomarkers most associated with prognosis ?**

## Objectives of the work

### COVID-19 case study

- Develop a multivariate joint model and a strategy to select a subset of biomarkers to predict the death of patients hospitalized for SARS-CoV-2 infection

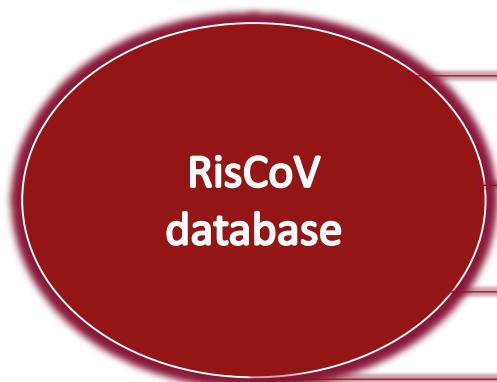
### Methodological assessment

- Evaluate the SAEM algorithm implemented in Monolix for multivariate joint models under competing risks
- Assess the validity of the proposed selection strategy

### *saemix* extension

- Extend the R *saemix*<sup>14</sup> package to the case of multi-response and joint models



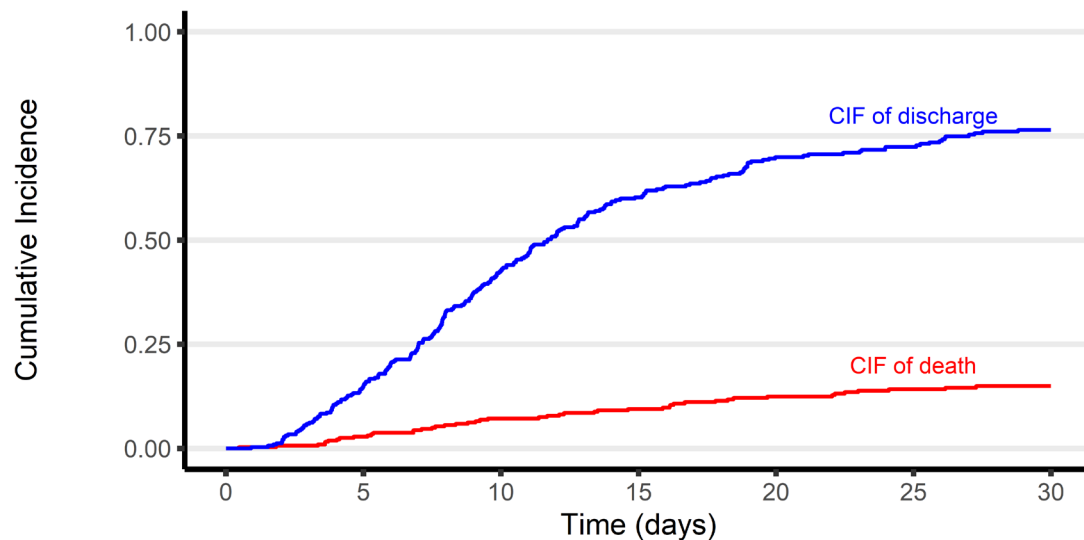
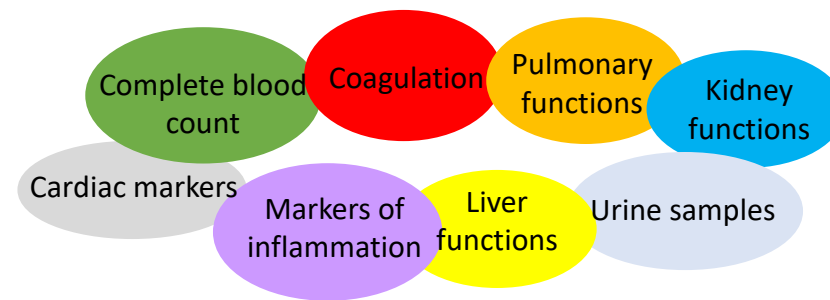


327 patients hospitalized in France during the first wave of the COVID-19 pandemic (January to July 2020)

59 biomarkers (classified in 8 categories) followed until death or discharge

4C Score<sup>15</sup> available at admission

At D30: 14% deaths  
73% discharges



Cumulative incidence functions for both events

Components of the 4C-Score

Age, years	< 50		0		
	50 – 59		+2		
	60 – 69		+4		
	70 – 79		+6		
	≥ 80		+7		
Sex at birth	Female	0	Male	+1	
Number of comorbidities	0	0	1	+1	
			≥ 2	+2	
Respiratory rate, breaths/min	< 20	0	20 – 29	+1	
			≥ 30	+2	
Peripheral oxygen saturation on room air	≥ 92%		0	< 92%	+2
	Glasgow Coma Scale		15	0	< 15
Urea (mmol/L) at admission	< 7	0	7 – 14	+1	
			> 14	+3	
C-reactive protein (mg/L) at admission	< 50	0	50 – 100	+1	
			≥ 100	+2	

## General notations

$y_{ijk}$ : obs of marker  $k$  in patient  $i$  at time  $t_{ijk}$   
 $Score_i$ : baseline 4C-Score for patient  $i$

$$y_{ijk} = \mathbf{m}_k(\boldsymbol{\psi}_{ik}, t_{ijk}) + g[m_k(\boldsymbol{\psi}_{ik}, t_{ijk}), \sigma_k] \varepsilon_{ij} \longrightarrow \text{Mixed-effects model}$$

### Subdistribution parametrization

$$h_{1ik}(t) = h_{01k} \times \exp(\alpha_{1k} \times \mathbf{m}_k(\boldsymbol{\psi}_{ik}, t) + \beta_{1k} \times Score_i) \longrightarrow \text{instantaneous risk of in-hospital death}$$

$$h_{2ik}(t) = h_{02k} \times \exp(\alpha_{2k} \times \mathbf{m}_k(\boldsymbol{\psi}_{ik}, t) + \beta_{2k} \times Score_i) \longrightarrow \text{instantaneous risk of discharge from hospital}$$

Linear model:

$$\mathbf{m}_k(\boldsymbol{\psi}_{ik}, t_{ijk}) = \psi_{0ik} + \psi_{1ik} \times t_{ijk}$$

Nonlinear model:

$$\mathbf{m}_k(\boldsymbol{\psi}_{ik}, t_{ijk}) = \psi_{0ik} + \psi_{aik} \times [\exp(\psi_{1ik} \times t_{ijk}) - \exp(\psi_{2ik} \times t_{ijk})]$$

$$\psi_{.ik} = \mu_{.k} + \eta_{.ik}$$

$$\psi_{aik} = \mu_{ak} \times \exp(\eta_{aik})$$

$$\eta_{.ik} \sim \mathcal{N}(0, \Omega_k)$$

$$\varepsilon_{ij} \sim \mathcal{N}(0, 1)$$

## General notations

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 $Score_i$ : baseline 4C-Score for patient  $i$   
 $K$ : number of biomarkers involved

$$y_{ij1} = \mathbf{m}_1(\psi_{i1}, t_{ij1}) + g[m_1(\psi_{i1}, t_{ij1}), \sigma_1] \varepsilon_{ij}$$

...

$$y_{ijK} = \mathbf{m}_K(\psi_{iK}, t_{ijK}) + g[m_K(\psi_{iK}, t_{ijK}), \sigma_K] \varepsilon_{ij}$$

$$h_{1i}(t) = h_{01} \times \exp(\alpha_{11} \times \mathbf{m}_1(\psi_{i1}, t) + \dots + \alpha_{1K} \times \mathbf{m}_K(\psi_{iK}, t) + \beta_1 \times Score_i)$$

$$h_{2i}(t) = h_{02} \times \exp(\alpha_{21} \times \mathbf{m}_1(\psi_{i1}, t) + \dots + \alpha_{2K} \times \mathbf{m}_K(\psi_{iK}, t) + \beta_2 \times Score_i)$$

Linear model:

$$\mathbf{m}_k(\psi_{ik}, t_{ijk}) = \psi_{0ik} + \psi_{1ik} \times t_{ijk}$$

Nonlinear model:

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...

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### Estimation

$$\theta = (\mu, \Omega, \sigma, h_{01}, h_{02}, \alpha_1, \alpha_2, \beta_1, \beta_2)$$

Monolix software version 2018R2



$$\mu = (\mu_1, \dots, \mu_K)$$

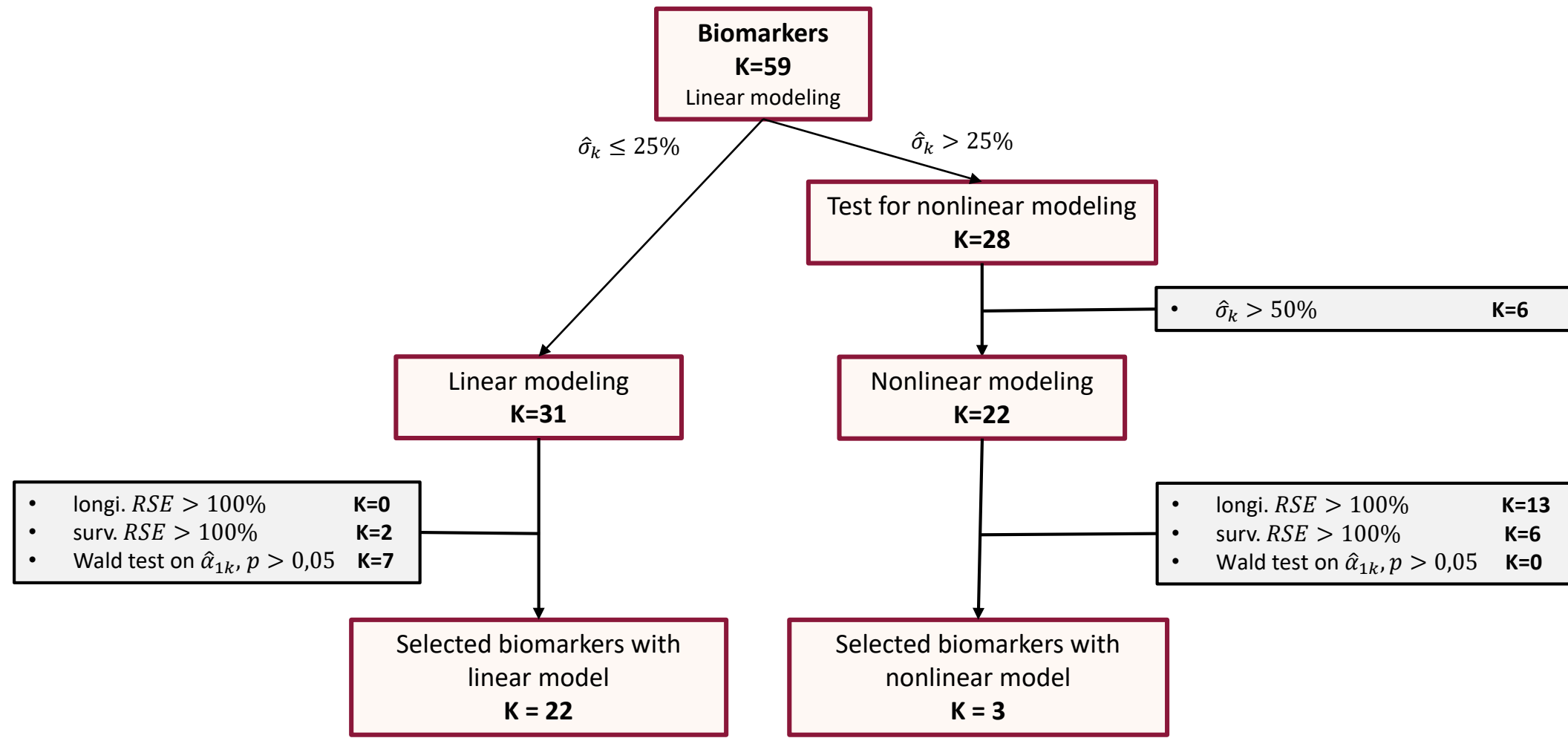
$$\Omega = \text{diag}(\omega_1, \dots, \omega_K)$$

$$\sigma = (\sigma_1, \dots, \sigma_K)$$

$$\alpha_1 = (\alpha_{11}, \dots, \alpha_{1K})$$

$$\alpha_2 = (\alpha_{21}, \dots, \alpha_{2K})$$

# Univariate joint models



## Univariate joint models

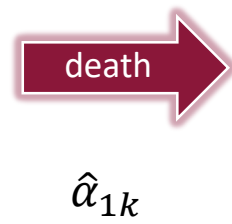
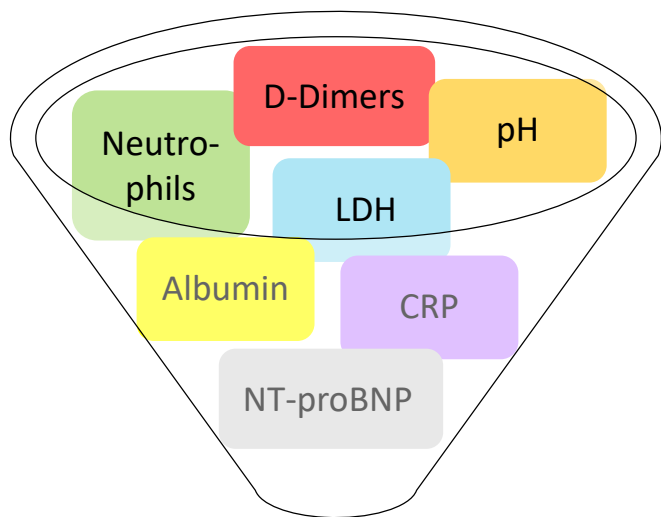
Biomarker	$\alpha_1$	RSE ( $\alpha_1$ )	$-\log_{10}(p)$	model
<b>Complete blood count</b>				
Neutrophil polynuclear cells	<b>0.24</b>	<b>16.17</b>	<b>9.20</b>	<b>nonlin</b>
Platelets	-0.004	27.66	3.52	lin
Erythrocytes	-0.44	45.28	1.57	lin
Hemoglobin	-0.14	49.06	1.38	lin
<b>Coagulation</b>				
<b>D-Dimers</b>	<b>1.08</b>	<b>14.86</b>	<b>10.78</b>	<b>lin</b>
Activated facteur V	0.04	18.40	7.26	lin
aPTT	1.50	20.00	6.24	lin
Fibrinogen	0.70	22.10	5.22	lin
Activated facteur II	-0.02	45.62	1.55	lin
<b>Pulmonary functions</b>				
<b>pHa</b>	<b>-20.61</b>	<b>11.18</b>	<b>18.42</b>	<b>lin</b>
pCO2a	0.19	12.48	14.95	lin
Oxyhemoglobin ratio	-2.04	45.23	1.57	lin
<b>Markers of inflammation</b>				
<b>CRP</b>	<b>1.25</b>	<b>17.63</b>	<b>7.85</b>	<b>lin</b>
Haptoglobin	0.42	19.03	6.83	lin
Orosomuroid	1.85	19.87	6.32	lin

Biomarker	$\alpha_1$	RSE ( $\alpha_1$ )	$-\log_{10}(p)$	model
<b>Blood kidney functions/cellular lysis</b>				
<b>Lactate deshydrogenase (LDH)</b>	<b>0.01</b>	<b>12.97</b>	<b>13.90</b>	<b>lin</b>
Uremia	0.07	18.11	7.48	nonlin
Kaliuresis	0.10	19.19	6.73	nonlin
Magnesium	6.65	23.96	4.52	lin
Calcemia	-6.00	25.27	4.12	lin
Creatininemia	0.003	32.27	2.71	lin
Phosphates	1.92	39.10	1.98	lin
Kalemia	0.99	44.01	1.64	lin
<b>Urine kidney functions</b>				
<b>Liver/pancreatic functions</b>				
<b>Albuminemia</b>	<b>-0.11</b>	<b>27.43</b>	<b>3.57</b>	<b>lin</b>
Lipasemia	0.88	17.22	8.19	lin
<b>Cardiac markers</b>				
<b>NT-proBNP</b>	<b>0.48</b>	<b>23.01</b>	<b>4.86</b>	<b>lin</b>

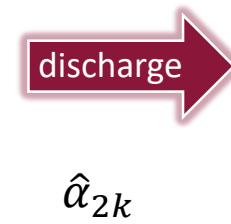
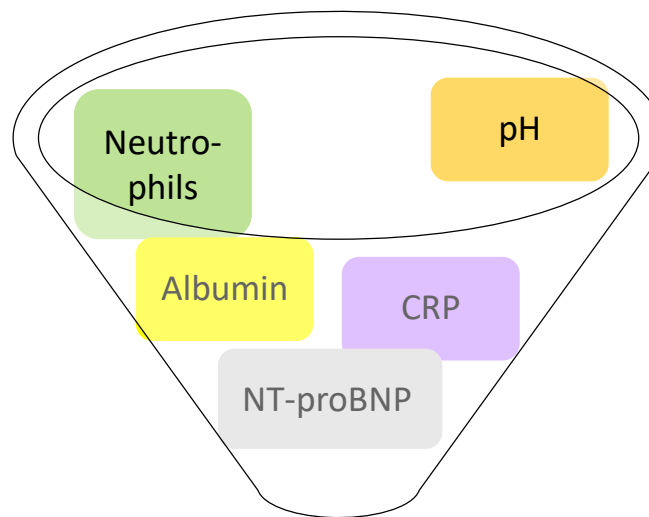


### Multivariate joint models

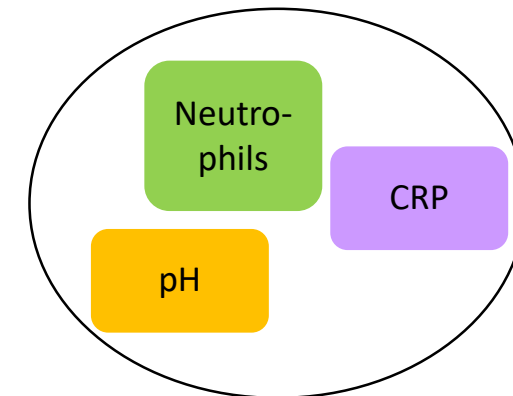
Initial multivariate joint model



Intermediate multivariate joint model



Final multivariate joint model

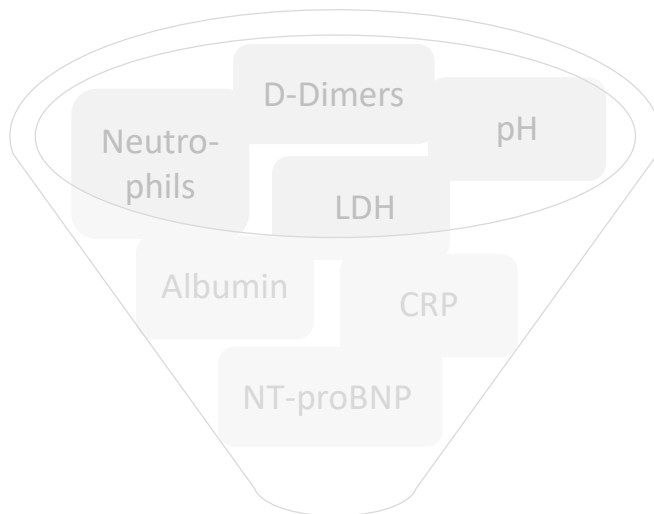


#### Backward selections

- Removed the highest p-value Wald test for  $\hat{\alpha}_{.k}$
- Stop when all p-values for  $< 5\%$

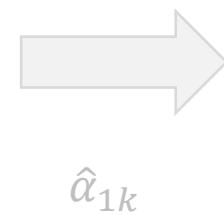
## Multivariate joint models

Initial multivariate joint model

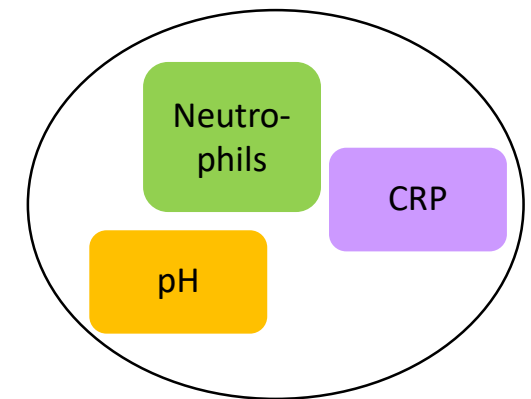


Intermediate multivariate joint model

Parameter	Value	R.S.E.(%)	p-value
<u>Death</u>			
$\alpha_{1neut}$ (L $\times 10^9$ )	0.14	24.2	< 10 <sup>-4</sup>
$\alpha_{1pH}$	-11.4	24.9	< 10 <sup>-4</sup>
$\alpha_{1CRP}$ (L.mg <sup>-1</sup> )	0.63	34.7	0.004
$\beta_1$	0.33	18.1	< 10 <sup>-4</sup>
<u>Discharge</u>			
$\alpha_{2neut}$ (L $\times 10^9$ )	-0.14	39.1	0.01
$\alpha_{2pH}$	25.2	25.2	< 10 <sup>-4</sup>
$\alpha_{2CRP}$ (L.mg <sup>-1</sup> )	-1.09	16.2	< 10 <sup>-4</sup>
$\beta_2$	-0.12	27.3	0.0002



Final multivariate joint model



- **Link and covariate parameter estimates (final multivariate joint model)**
- stop when all p-values for < 5%

# Multivariate joint models

## Initial multivariate joint model

## Intermediate multivariate joint model

## Final multivariate joint model

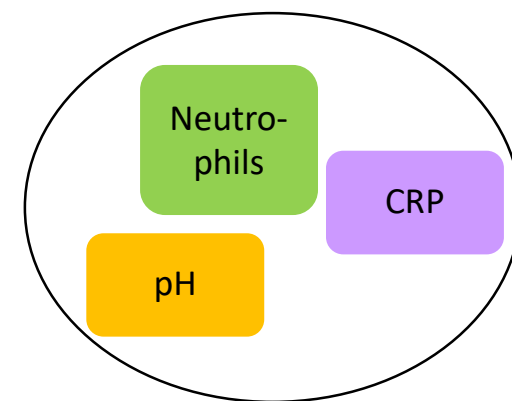
From the final multivariate joint model:

- Derivation of individual dynamic predictions
- Good prediction performances
- Better than a model that only consider baseline information

- 30th PAGE conference (2022), " Longitudinal biomarkers predicting death of hospitalized patients for SARS-COV-2 infection : a joint analysis with competing risks "

### Backward selections

- Removed the highest p-value Wald test for  $\hat{\alpha}_{2k}$
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## Objectives

- Evaluate the performances of the estimation
- Assess the validity of the proposed selection strategy

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**Data generating mechanism**

$M = 100$  datasets of  $N = 300$  patients

$K = 7$  biomarkers ( $bm_1$  to  $bm_7$ ) simulated according to the design of the application (multivariate stage)

2 failure causes: death (event 1) and discharge (event 2)

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Biomarker	Longitudinal submodel	Error model	Measurement frequency	Association with event 1	Correlation on slopes
$bm_1$	Nonlinear	Proportional	Every 2 days	✓	
$bm_2$	Linear	Additive	Every 1.5 days	✓	
$bm_3$	Linear	Additive	Every 2 days	✓	
$bm_4$	Linear	Additive	Every 3 days		$\rho(\eta_{.4}, \eta_{.2}) = 0.8$
$bm_5$	Linear	Proportional	Every 3 days		$\rho(\eta_{.5}, \eta_{.3}) = 0.8$
$bm_6$	Linear	Proportional	Every 3 days		
$bm_7$	Linear	Proportional	Every 3 days		

$$h_{i1}(t, \psi_i; \theta) = \frac{p_1 g_1 \exp(-g_1 \times t)}{1 - p_1 (1 - \exp(-g_1 \times t))} \exp(\alpha_{11} \times m_1(\psi_{i1}, t) + \dots + \alpha_{13} \times m_3(\psi_{i3}, t))$$

$$h_{i2}(t, \psi_i; \theta) = \frac{1}{b} \times \frac{(1 - F_1(\infty)) \exp(-t/b)}{1 - (1 - F_1(\infty))(1 - \exp(-t/b))}$$

## Estimands and performances measures

### Objective 1: assess the performances of the estimation

For a simulation  $m \in \{1, \dots, M\}$ :

- Estimation of  $\theta$  (true model parameters)
- Performances assessed with relative estimation errors:

$$\text{REE}^m(\hat{\theta}) = \frac{\hat{\theta}^m - \theta}{\theta} \times 100$$

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### Objective 2: assess the ability of the backward strategy to find the “true” model

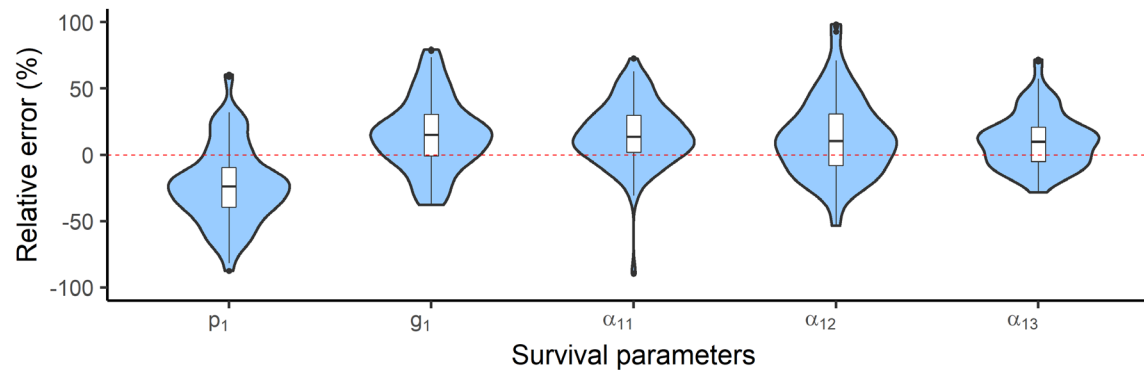
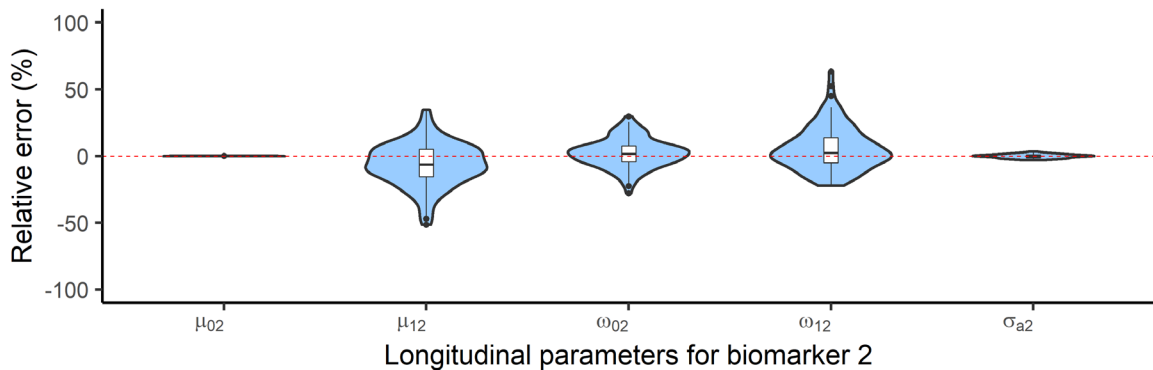
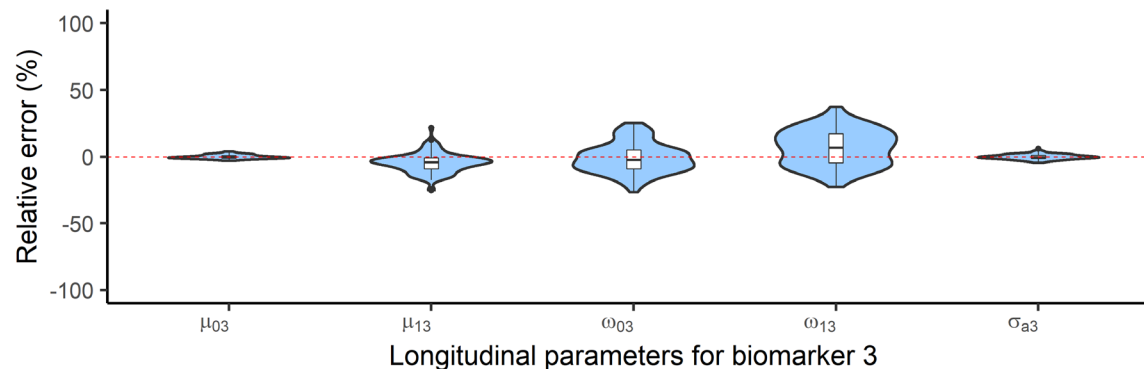
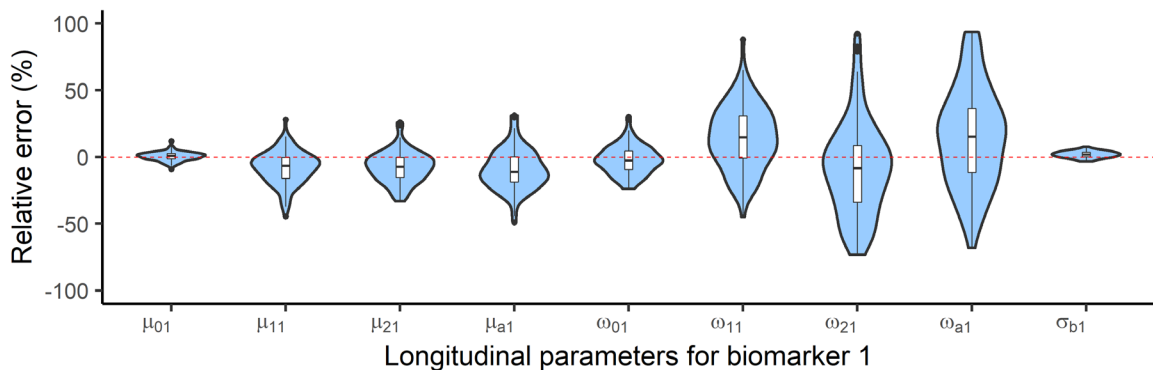
For a simulation  $m \in \{1, \dots, M\}$ :

- Start with the full multivariate model (7 biomarkers)
- Backward process on  $\hat{\alpha}_{1k}$  (stop when all p-values are  $< 5\%$ )
- Performances assessed by reporting the final set of biomarkers



# Results

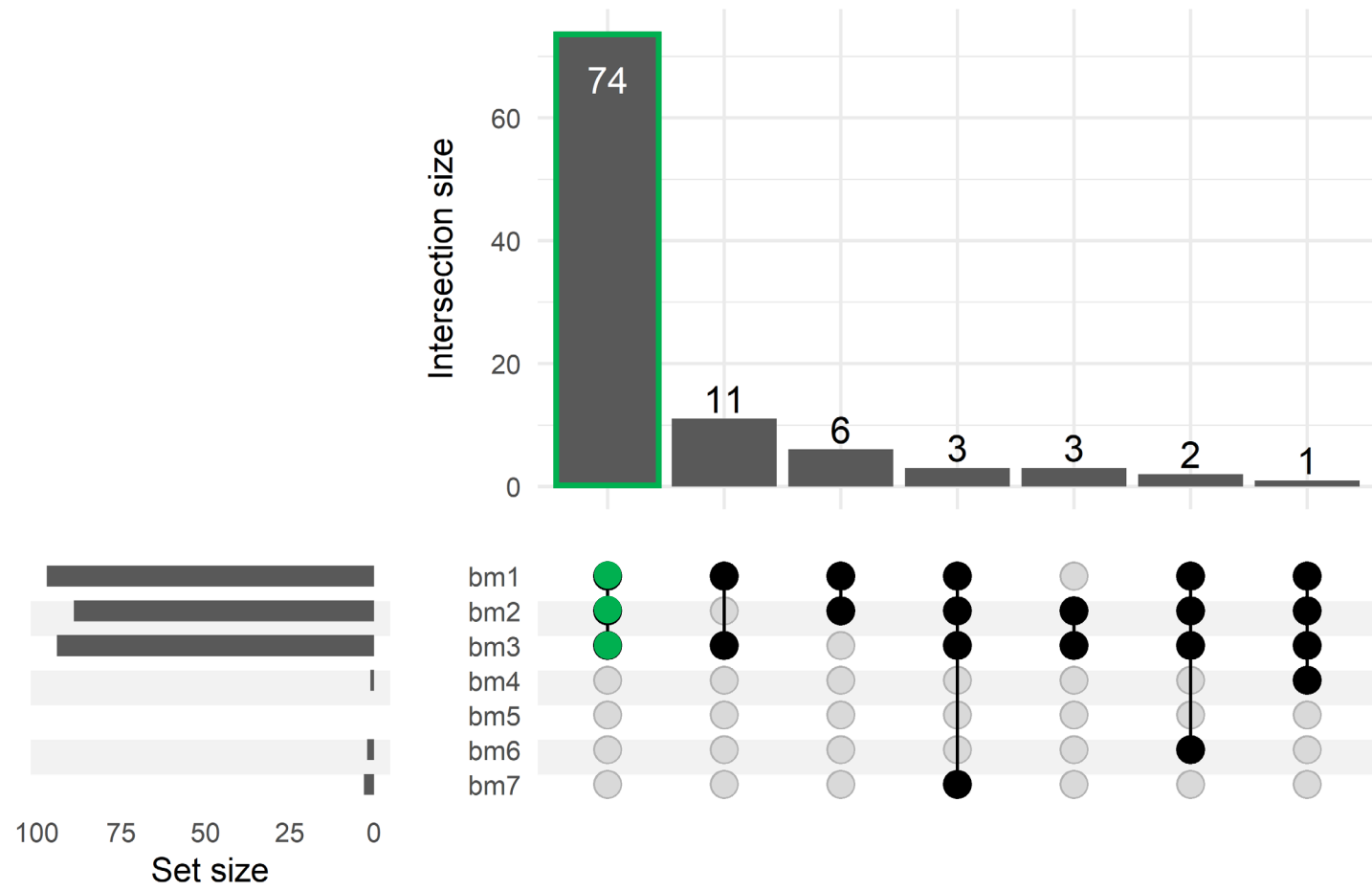
**Objective 1: assess the performances of the estimation**



Distribution of the Relative Estimation Errors for each model parameter

# Results

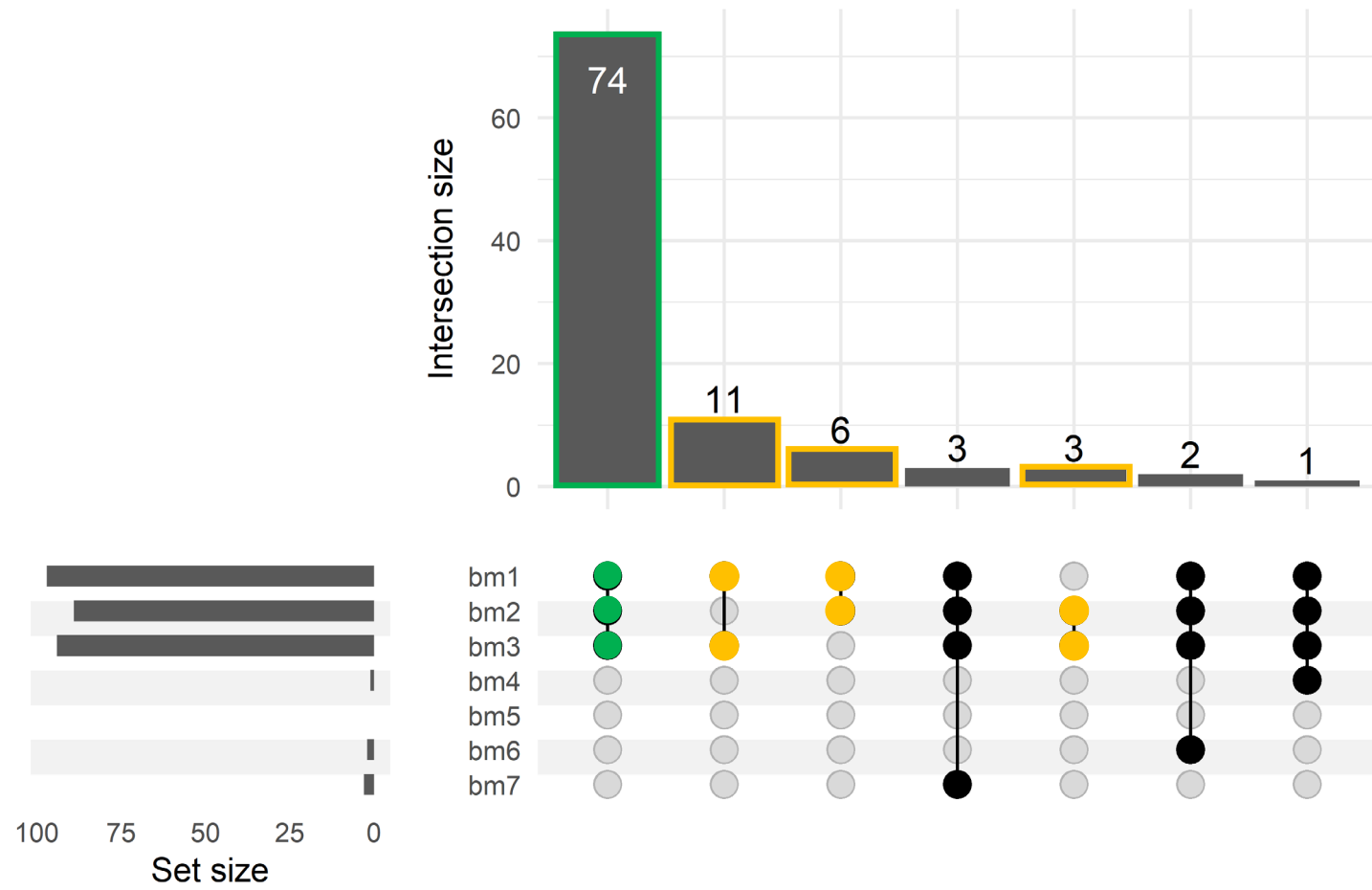
**Objective 2: assess the ability of the backward strategy to find the “true” model**



Final set of biomarkers selected after the backward process for each simulation

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  - Alternative stochastic algorithm developed by Delattre and Kuhn<sup>17</sup>

➡ interesting properties because it is integrated in the SAEM algorithm (time saving)

➡ but not currently evaluated for joint models

16- Louis, *Journal of the Royal Statistical Society*, 1982

17- Delattre and Kuhn, *hal-02285712v2*, 2023

## Evaluation by simulations

### Aims

1. Evaluate the SAEM algorithm extended in R *saemix* package<sup>14</sup> for joint models:

	Single event	2 competing risks
Linear mixed-effects model	LMEM – TTE	LMEM – CR
Nonlinear mixed-effects model	NLMEM – TTE	NLMEM – CR

2. Evaluate the algorithm developed by Delattre and Kuhn<sup>17</sup> for the 4 previous models (standard errors estimation)

## Evaluation by simulations

### Data generating mechanism

For each of the 4 models presented, we simulate  $M = 100$  datasets of  $N = 100$  patients.  
Biomarker measurements available each day until time-to-event for at most 30 days

#### LMEM – TTE

$$y_{ij} = m_l(\psi_i, t_{ij}) + \sigma \epsilon_{ij}$$

$$h_i(t, \psi_i; \theta) = h_0 \times \exp(\alpha \times m_l(\psi_i, t))$$

#### LMEM-CR

$$y_{ij} = m_l(\psi_i, t_{ij}) + \sigma \epsilon_{ij}$$

$$h_{i1}(t, \psi_i; \theta) = \frac{p_1 g_1 \exp(-g_1 \times t)}{1 - p_1 (1 - \exp(-g_1 \times t))} \exp(\alpha_1 \times m_l(\psi_i, t))$$

$$h_{i2}(t, \psi_i; \theta) = \frac{1}{b} \times \frac{(1 - F_1(\infty)) \exp(-t/b)}{1 - (1 - F_1(\infty))(1 - \exp(-t/b))}$$

#### NLMEM-TTE

$$y_{ij} = m_{nl}(\psi_i, t_{ij}) + \sigma \epsilon_{ij}$$

$$h_i(t, \psi_i; \theta) = h_0 \times \exp(\alpha \times m_{nl}(\psi_i, t))$$

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## Evaluation by simulations

### Estimands

#### Objective 1: assess parameter estimation

JM LMEM/NLMEM – TTE

$$\theta = (\mu, \Omega, \sigma, h_0, \alpha)$$

JM LMEM/NLMEM – CR

$$\theta = (\mu, \Omega, \sigma, p_1, g_1, \alpha, b)$$



### Performance measures

Relative estimation errors:

$$\text{REE}^m(\hat{\theta}) = \frac{\hat{\theta}^m - \theta}{\theta} \times 100$$

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#### Objective 2: assess standard error estimation

JM LMEM/NLMEM – TTE/CR

$\hat{\Sigma}$  = variance-covariance matrix of  $\hat{\theta}$



### Performance measures

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$$\text{REE}^m(\hat{\theta}) = \frac{\hat{\theta}^m - \theta}{\theta} \times 100$$

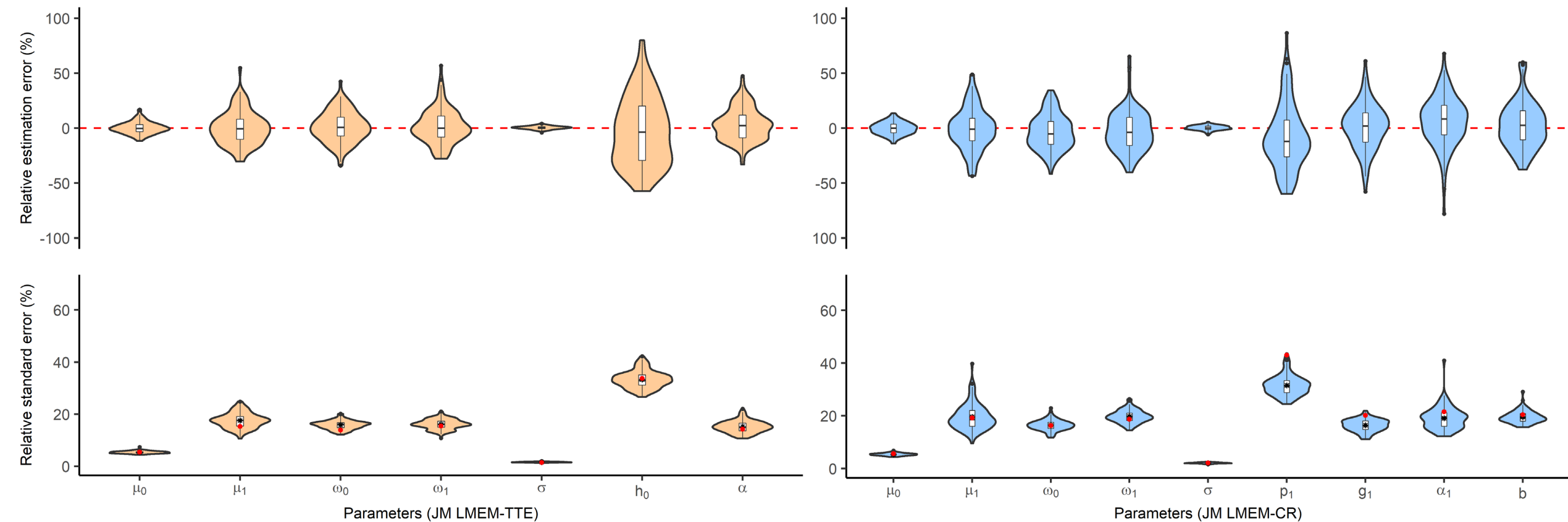
Relative standard errors:

$$\text{RSE}^m(\hat{\theta}) = \frac{\sqrt{\text{diag}(\hat{\Sigma}^m)}}{\hat{\theta}^m} \times 100$$

Relative empirical error:

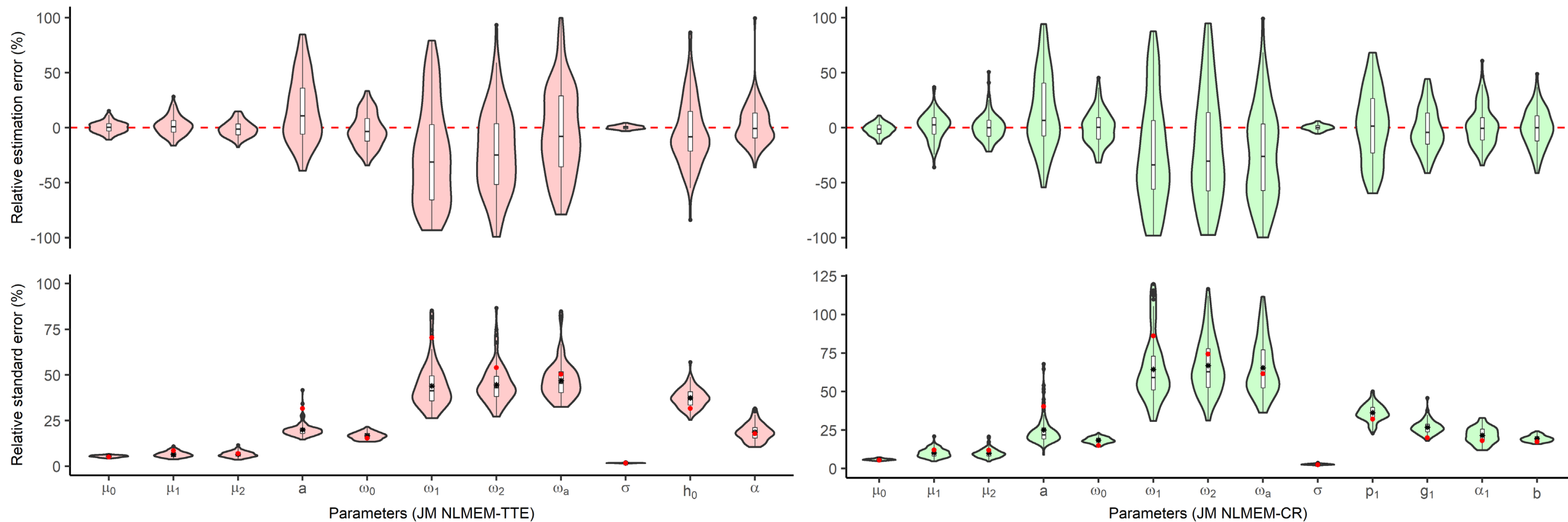
$$\text{RSE}^{\text{emp}}(\hat{\theta}) = \sqrt{\frac{1}{m-1} \sum_{m=1}^M (\hat{\theta}^m - \bar{\hat{\theta}})^2} \times \frac{100}{\bar{\hat{\theta}}}$$

### Distribution of REE (top) and stochastic RSE versus empirical RSE (bottom) - JM LMEM-TTE and JM LMEM-CR



- Empirical RSE
- ★ Mean distribution of the stochastic RSE

## Distribution of REE (top) and stochastic RSE versus empirical RSE (bottom) - JM NLMEM-TTE and JM NLMEM-CR



- Empirical RSE
- ★ Mean distribution of the stochastic RSE

## COVID-19 case study

- Developments of a multivariate joint model to predict the death of patients hospitalized for SARS-CoV-2 infection and a strategy to select among various biomarkers
- Evolution of neutrophils, pH and CRP are predictive of the death/discharge of patients
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- The backward strategy yields good performances

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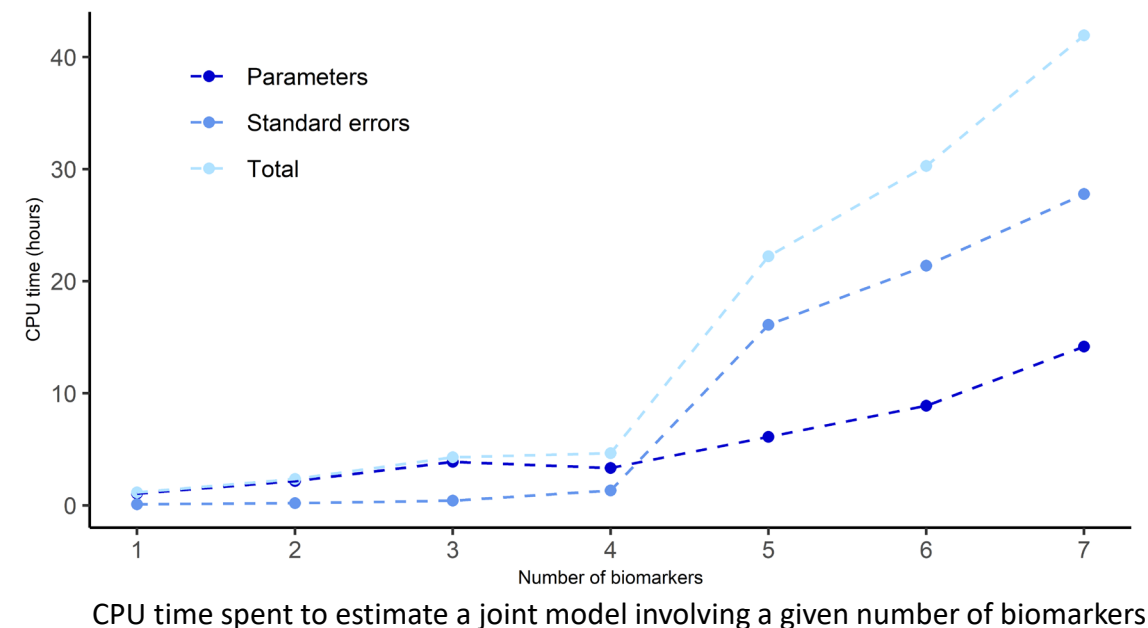
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- The backward strategy yields good performances

### Limitations

- Limited number of biomarkers included in the multivariate analysis (computational limit)
- Backward strategy usually outperformed by penalized regression methods<sup>18</sup> (LASSO penalization)



## *saemix* extension

- Extension of the R package *saemix* to the case of joint models
- Good properties for parameter and standard errors estimation
- Flexible tool in parametric joint model framework
- Users define the likelihood of the model (very specific joint models can be considered)

### Perspectives

- Need to evaluate for multiple longitudinal biomarkers
- Need to develop goodness of fit tools in the package

Functions and examples available on Github:  
<https://github.com/saemixdevelopment/saemixextension/tree/master/joint>





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**Thank you for your attention !**

